



Level 2 Certificate
FURTHER MATHEMATICS
8365/1

Paper 1 Non-Calculator

Mark scheme

June 2024

Version: 1.0 Final



2 4 6 G 8 3 6 5 / 1 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

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Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
3.14...	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416

Examiners should consistently apply the following principles.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Continental notation

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

MARK SCHEME – LEVEL 2 CERTIFICATE FURTHER MATHEMATICS – 8365/1 – JUNE 2024

Q	Answer	Mark	Comments
1	2420 or 121 or 1.21×10^2	M1	doesn't need to be seen in a correct expression
	11	A1	accept ± 11
	Additional Guidance		
	Condone $t = 11$		M1A1
	Condone 1.1×10^1 or 1.1×10		M1A1
	$\frac{1210}{10}$ or $\frac{242}{2}$		M1
	$\frac{\sqrt{121}}{10}$ whilst incorrect would gain the method mark for 121		M1

Q	Answer	Mark	Comments
2	$(x + y)(x - y)$	B1	either order
	Additional Guidance		
	Do not condone missing brackets		
	Penalise further working on answer line		
	$xy(xy^{-1} - x^{-1}y)$		B0
	$(-x + y)(-x - y)$		B1

Q	Answer	Mark	Comments
3	3	B1	
	Additional Guidance		

Q	Answer	Mark	Comments
4	Alternative method 1		
	$y = 3x + 4$ from first line or $y = 3x - \frac{5}{6}$ from second line	M1	oe where one equation is rearranged correctly to match the other eg $6y = 18x + 24$ from first line
	$y = 3x + 4$ and $y = 3x - \frac{5}{6}$ and a statement that they have the same gradient so are parallel	A1	allow $y - 3x = 4$ and $y - 3x = -\frac{5}{6}$ eg same gradient so parallel same gradient but different y intercept so parallel $m_1 = m_2$ so parallel
	Alternative method 2		
	2 correct points on at least one line found and at least one correct calculation using $\frac{y_2 - y_1}{x_2 - x_1}$ to get $m = 3$	M1	eg (1, 7) & (2, 10) or $(0, -\frac{5}{6})$ & $(1, \frac{13}{6})$ and $\frac{10-7}{2-1}$ or $\frac{13-\frac{5}{6}}{1-0}$ or $\frac{\text{rise}}{\text{run}} = \frac{3}{1}$
	$m = 3$ for both lines calculated using $\frac{y_2 - y_1}{x_2 - x_1}$ and a statement that they have the same gradient so are parallel	A1	eg. same gradient so parallel same gradient but different y intercept so parallel $m_1 = m_2$ so parallel must see two separate calculations
	Additional Guidance		
	In Alt 1 both equations must be correctly rearranged for A mark		
	Condone $3x$ as gradient rather than 3		
	Gradient = 18 (must rearrange to get gradient of 3 to gain A mark in Alt 1)		A0
Condone m or $\frac{dy}{dx}$ used for gradient			
A statement that only mentions gradient and doesn't say they are parallel		A0	

Q	Answer	Mark	Comments	
5	$4x^{-1}$ or $\frac{4}{x}$ or x^3	M1	either term correct (ignore additional terms)	
	$-4x^{-2}$ or $3x^2$	A1	a correct term differentiated correctly (must be just two terms before differentiation) accept $-\frac{4}{x^2}$	
	$\left(\frac{dy}{dx} =\right) -4x^{-2} + 3x^2$	A1	both terms correct accept $-\frac{4}{x^2}$	
	Additional Guidance			
	Quotient rule or product rule used to achieve correct simplified answer	M1A2		
	Quotient rule or product rule used with incorrect or unsimplified answer	M0A0		
	$\frac{-4 + 3x^4}{x^2}$	M1A2		
Penalise additional incorrect further working eg $\frac{-1}{4x^2}$ for the final A mark				

Q	Answer	Mark	Comments		
6	$x^2 + y^2 = 145$	B3	B2 centre (0, 0) and (radius =) $\sqrt{145}$ or centre (0, 0) and (radius ² =) 145 or $(x - 0)^2 + (y - 0)^2 = 145$ or $x^2 + y^2 = \sqrt{145}$ B1 $x^2 + y^2 = k$ or $(x - 0)^2 + (y - 0)^2 = k$ or $(x + \dots)^2 + (y + \dots)^2 = 145$ or centre (0, 0) or (radius =) $\sqrt{145}$ or (radius =) $\sqrt{12^2 + 1^2}$ or (radius ² =) 145 or $12^2 + 1^2$ or (radius =) $\frac{\sqrt{580}}{2}$ or (radius ² =) $\frac{580}{4}$		
			Additional Guidance		
			k is a positive number (not 0)		
			Penalise further incorrect working		
			(0, 0) needs to be stated as centre and not just shown on a diagram nor stated as a midpoint		
No marks for finding diameter on its own					

Q	Answer	Mark	Comments	
7	$\begin{pmatrix} 4 & 0 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ 7 \end{pmatrix}$	M1	may be inferred from correct second M	
	$4x = -8$ and $-2x + 3y = 7$	M1dep	oe may be seen in vector notation	
	$(x =) -2$ and $(y =) 1$	A1		
	Additional Guidance			
	$x = -2$ if no other marks scored	SC1		
	$\begin{pmatrix} 4 & 0 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} -8 \\ 7 \end{pmatrix} = \begin{pmatrix} -32 \\ 37 \end{pmatrix}$ or $(x =) -32$ and $(y =) 37$	SC1		
Condone $\begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} 4 & 0 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} -8 \\ 7 \end{pmatrix}$ if recovered to get correct equations				
Condone answer left as column vector $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ or as coordinates $(-2, 1)$				

Q	Answer	Mark	Comments
8	$x(2x^2 - 9x - 5)$ or $(2x^2 + x)(x - 5)$ or $(2x + 1)(x^2 - 5x)$ or states $x = 0$ and $(2x^2 - 9x - 5)$	M1	
	$x(2x + 1)(x - 5)$ or states $x = 0$ and $(2x + 1)(x - 5)$	M1dep	do not accept $x(2x + 1)(2x - 10)$ unless recovered by correct solutions
	$(x =) 0, 5, -0.5$	A1	oe but must come from factorising in M1dep
	Additional Guidance		
	$(2x + 1)(x - 5)$ if no other marks scored		SC1
$(x =) 0, 5, -0.5$ without working or by using factor theorem to find solutions without writing it as a product of factors or by using quadratic formula (must be all 3 correct answers)		SC1	

Q	Answer	Mark	Comments
9	$3^2 + 4^2$ or $3^2 + 12^2$ or $4^2 + 12^2$ or (EG \Rightarrow) 5	M1	oe could be seen as $\sqrt{\quad}$ may be seen evaluated eg. 25, 153 or 160 may be seen on diagram
	$3^2 + 4^2 + 12^2$ or $5^2 + 12^2$ or 169	M1dep	oe could be seen as $\sqrt{\quad}$ must be expressions that could be evaluated to give 169
	13	A1	
	Additional Guidance		
	$4\sqrt{11}$ or $2\sqrt{44}$ or $\sqrt{176}$ or $\sqrt{162}$ or $3\sqrt{18}$ or $9\sqrt{2}$		SC1

Q	Answer	Mark	Comments	
10	P is (2, 20) or y value of P is 20	M1	implied by $4a + 2b = 20$ could be seen on diagram	
	$4a + 2b = k$	M1	oe $4a + 2b = 20$ k could be any number (including 0) but must be 20 if first M mark awarded	
	$36a + 6b = 12$	M1	oe	
	$a = -2$ or $b = 14$	A1		
	$a = -2$ and $b = 14$	A1		
	Additional Guidance			
	$a = -2$ and $b = 14$ with no working			M3A2
	$a = -2$ or $b = 14$ with no working			M0A0

Q	Answer	Mark	Comments
11(a)	Alternative method 1 – by inspection or long division		
	$x^2 + kx - 2$ by inspection or first two terms using long division: $x^2 + x + k$	M1	k could be blank can be seen in a grid method
	$x^2 + x - 2$	A1	
	$(x - 1)$ and $(x + 2)$ or $(x - 1)(x + 2)$ or $(3x - 7)(x - 1)(x + 2)$	A1	any order of brackets
	Alternative method 2 – factor theorem		
	1 or 2 or -1 or -2 substituted into function and evaluated correctly	M1	$f(x) = 0$ for $x = 1$ $f(x) = 0$ for $x = -2$ $f(x) = 20$ for $x = -1$ $f(x) = -4$ for $x = 2$ ignore additional incorrect calculations
	$(x + 2)$ or $(x - 1)$ identified as a factor	A1	must have come from a substitution from M mark
	$(x - 1)$ and $(x + 2)$ or $(x - 1)(x + 2)$ or $(3x - 7)(x - 1)(x + 2)$	A1	any order of brackets
	Alternative method 3 – equating coefficients		
	$(3x - 7)(ax^2 + bx + c)$ and $3ax^3 - 7ax^2 + 3bx^2 - 7bx + 3cx - 7c$	M1	
	$a = 1, b = 1$ and $c = -2$ or $x^2 + x - 2$	A1	
	$(x - 1)$ and $(x + 2)$ or $(x - 1)(x + 2)$ or $(3x - 7)(x - 1)(x + 2)$	A1	any order of brackets
	Additional Guidance		
	$x = 1$ and -2 on answer line or last line of working		SC2
Follow whichever scheme gives the best mark			
Penalise incorrect further working eg. correct answer followed by $x = 1$ and $x = -2$		M1A1A0	

Q	Answer	Mark	Comments
11(b)	Alternative method 1 – by substitution		
	$a \times 2^4 - 3a \times 2^3 + 5 \times 2 - 22$ or $16a - 24a + 10 - 22$	M1	oe 4 terms with at least 3 correct
	$16a - 24a + 10 - 22 = 0$ or $-8a = 12$	M1dep	oe could imply first M mark
	$a = -1\frac{1}{2}$ or $\frac{-3}{2}$ or -1.5	A1	
	Alternative method 2 – long division		
	First two terms correct	M1	$ax^3 - ax^2$
	Correct equation set up to find a	M1dep	eg $-8a = 12$ or $5 - 4a = 11$
	$a = -1\frac{1}{2}$ or $\frac{-3}{2}$ or -1.5	A1	
	Alternative method 3 – equating coefficients or inspection		
	$d = 11$ and ax^3 and $c = 3$ or $b = 1.5$ or $c = -2a$ or $b = -a$	M1	could be seen in a grid for $ax^3 + bx^2 + cx + d$
	Correct equation set up to find a	M1dep	eg $4a + 11 = 5$ or $2a + 3 = 0$
	$a = -1\frac{1}{2}$ or $\frac{-3}{2}$ or -1.5	A1	
	Additional Guidance		
Follow whichever scheme gives the best mark			

Q	Answer	Mark	Comments
12	$12^2 + 10^2 - 2 \times 12 \times 10 \times \frac{3}{4}$	M1	oe in a fully correct substitution could already be under $\sqrt{\quad}$
	$144 + 100 - 180$ or 64	M1dep	
	8	A1	± 8 is A0
	Additional Guidance		
	$\cos \frac{3}{4}$ used in the working will not score unless recovered into a fully correct calculation that scores to at least the first M mark		

Q	Answer	Mark	Comments	
13	$7x + 8(x - 3)$	M1	oe eg $7 \times 3 + 15(x - 3)$ or $(8 + 7)x - 8 \times 3$	
	A correct inequality formed for x	M1dep	eg. $7x + 8x - 24 < 51$ or $7x + 8(x - 3) < 51$ or $15x < 75$ or $21 + 15x - 45 < 51$	
	$x < 5$	A1		
	$3 < x (< k)$	B1	k greater than 3 or blank on answer line	
	Additional Guidance			
	$3 < x < 5$ with no or incorrect working can only score the B1			
	Trial and Improvement: stop marking once this begins (may have scored M2 and may go on to score B1)			
	Ignore any units stated			
Inequalities replaced with = or other inequalities are penalised unless recovered				

Q	Answer	Mark	Comments
14(a)	Alternative method 1		
	$2(x^2 - 8x + \dots)$ $2(x - 4)^2 + \dots$	M1	oe
	$2[(x - 4)^2 - 4^2] + \dots$ or $2[(x - 4)^2 - 16] + \dots$ or $2[(x - 4)^2 - 16 + \frac{7}{2}]$ or $2[(x - 4)^2 - \frac{25}{2}]$ or $2(x - 4)^2 - 2 \times \frac{25}{2}$ or $2(x - 4)^2 - 32 + 7$	M1dep	oe the bracket is after the 4^2 and the 16 here. If they put something else inside the bracket it is incorrect unless it is equivalent to one of the fully complete versions listed
	$2(x - 4)^2 - 25$	A1	
	Alternative method 2 – using identities		
	$kx^2 + 2kmx + km^2 + n$	M1	RHS oe
	$k = 2$ and $2km = -16$ and $km^2 + n = 7$	M1dep	oe
	$2(x - 4)^2 - 25$	A1	
	Additional Guidance		
	Condone answer followed by $k = 2$ $m = -4$ $n = -25$		M2A1
$k = 2$ $m = -4$ $n = -25$ without correct answer is penalised		M2A0	

Q	Answer	Mark	Comments
14(b)	$(x =) 1 \pm \sqrt{5}$	B1	
	Additional Guidance		
	Condone roots written separately (and/or are both fine)		
	Do not condone $1 + \sqrt{5}$ on its own or $1 - \sqrt{5}$ on its own		
Do not accept oe answers – must be simplified			

Q	Answer	Mark	Comments
15(a)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1	
	Additional Guidance		

Q	Answer	Mark	Comments
15(b)	$(\mathbf{N}^2 =) \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ or describes \mathbf{N} as a rotation 90° clockwise about the origin (or (0, 0))	M1	can be implied from A1 condone rotation of -90° or 270° about the origin (or (0, 0))
	Rotation 180° about the origin (or (0, 0)) or Enlargement sf. -1 about the origin (or (0, 0))	A1	clockwise or anticlockwise can be stated
	Additional Guidance		
	$(\mathbf{N} =)$ Rotation 90° (without clockwise) about the origin recovered with a correct answer		M1A1
	$(\mathbf{N} =)$ Rotation 90° anticlockwise about the origin with correct answer would be from incorrect working		M0A0
	$(\mathbf{N} =)$ Rotation 90° (without clockwise) about the origin without the correct answer		M0A0
	Missing 'about the origin' in M mark can be recovered in A mark		
Condone missing brackets in matrices			

Q	Answer	Mark	Comments	
16	$(BOD =) 104^\circ$	B1		
	$(BCD =) 128^\circ$	B1		
	$(OBC + ODC =)$ $360^\circ - (\text{their})128^\circ - (\text{their})104^\circ$ or $OBC + ODC = 128^\circ$ or $128 = 8x$	M1	oe eg $360^\circ - 232^\circ$ one of 128° and 104° must be correct this could imply B1 B1 oe	
	$(OBC =) 80^\circ$	A1	can only be awarded with evidence of B2M1	
	Additional Guidance			
	Angles may be seen on diagram			
	Check they are not splitting BCD in the ratio 5:3 (benefit of doubt may need to be applied). It is possible to draw a line from O to C which splits the quadrilateral into two isosceles triangles. This method does allow for BCD to be split in the ratio 5:3			
	$OBC = 80^\circ$ and $ODC = 48^\circ$ both on answer line			A1
	80° and 48° both on answer line would be choice			A0
Condone other forms of angle notation				

Q	Answer	Mark	Comments
17	Alternative method 1		
	$(3x + 4)(x - 2)$	M1	oe for correct factorisation
	$\frac{21x - 7(3x + 4)}{(3x + 4)(x - 2)}$	M1dep	oe fractions can be written separately denominator must be $(3x + 4)(x - 2)$ or $3x^2 - 2x - 8$
	$\frac{-28}{3x^2 - 2x - 8}$	A1	
	Alternative method 2		
	$\frac{21x(x - 2) - 7(3x^2 - 2x - 8)}{(3x^2 - 2x - 8)(x - 2)}$	M1	oe fractions can be written separately
	$\frac{21x^2 - 42x - 21x^2 + 14x + 56}{(3x^2 - 2x - 8)(x - 2)}$	M1dep	oe must be one fraction and brackets expanded in the numerator (could have been simplified and factorised) eg $\frac{-28(x - 2)}{(3x^2 - 2x - 8)(x - 2)}$ expanded denominator is $3x^3 - 8x^2 - 4x + 16$
	$\frac{-28}{3x^2 - 2x - 8}$	A1	
	Additional Guidance		
	Students may use a combination of methods which are covered by oe		
$k = -28$ Could come from using identities or substitutions		SC1	
The minus can be written before the fraction rather than with the 28			

Q	Answer	Mark	Comments
18	$\frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}}$	M1	oe eg $\frac{3\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} \times \frac{-\sqrt{5} - \sqrt{3}}{-\sqrt{5} - \sqrt{3}}$
	Denominator $5 - 3$ or 2	M1dep	
	Numerator $15 + 3 + 3\sqrt{15} + \sqrt{15}$	M1dep	oe three terms correct could be simplified without writing 4 terms which would imply original terms if done correctly eg $18 + 2\sqrt{15} + \sqrt{15}$ or $18 + 3\sqrt{15}$ whereas $17 + 3\sqrt{15} + \sqrt{15}$ could be two errors so wouldn't get M1 accept $\sqrt{3}\sqrt{5}$ for $\sqrt{15}$ for this mark do not accept $\sqrt{3}\sqrt{3}$ or $\sqrt{9}$ for 3 or $\sqrt{5}\sqrt{5}$ or $\sqrt{25}$ for 5 this would count as one error dep on first M mark only
	$9 + 2\sqrt{15}$	A1	
	Additional Guidance		
	For M dep marks numerator and denominator can be seen separately		
	Follow an equivalent MS for any student using $\frac{3\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} \times \frac{-\sqrt{5} - \sqrt{3}}{-\sqrt{5} - \sqrt{3}}$		
	M marks may be seen in a grid method		
Untidy mathematical notation can be condoned as long as it's recovered			
Missing brackets can be recovered			

Q	Answer	Mark	Comments
19	$\left(\frac{dy}{dx} = 0\right) 3x^2 - 6x$	M1	either term correct could have additional incorrect terms
	Their $\frac{dy}{dx} = 0$	M1dep	follow through an incorrect differentiation as long as M1 scored
	$x = 2$ and $x = 0$	A1	these values could imply M1dep if first M mark awarded
	$\left(\frac{d^2y}{dx^2} = 6x - 6\right)$ and $\left(\frac{d^2y}{dx^2} = 6\right)$ and/or positive (for $x = 2$) or $\left(\frac{d^2y}{dx^2} = -6\right)$ and/or negative (for $x = 0$) or $\left(\frac{dy}{dx} = 0\right) 3x^2 - 6x$ and any correct check for both sides of one correct solution to give one side with a negative gradient and one side with a positive gradient	M1dep	a correct x value assessed correctly eg $x = -1 \frac{dy}{dx} > 0$ and $x = 1 \frac{dy}{dx} < 0$ or $x = 1 \frac{dy}{dx} < 0$ and $x = 3 \frac{dy}{dx} > 0$
	(0, 5) Maximum (2, 1) Minimum	A1	both points must have been assessed correctly
	Additional Guidance		
	Final A mark must have everything correct including both stationary points assessed correctly		
	Just a sketch is not enough to determine the nature of the stationary points		
Condone incorrect writing of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ even if it's just $y =$ as long as it's recovered to get the correct nature of the turning points			

Q	Answer	Mark	Comments
	$(y =) \frac{-x}{2} + 3$	B2	oe eg $2y = -x + 6$ can be implied from correct line B1 for $(y =) \frac{x}{2} - 3$ can be implied from drawing this line or $m = -\frac{1}{2}$ or $c = +3$ not from incorrect working for either m or c
20	$y = \frac{-x}{2} + 3$ accurately drawn on diagram	M1	ft their stated linear function in x accuracy is half a square at $x = 0$ and $x = 4$ needs to be a straight line (mark intention)
	[-0.6, -0.8] and [4.1, 4.3]	A1	dependent on B2M1
Additional Guidance			
Factorising by use of the quadratic formula alone			B0M0A0
Ignore any y values for A1			
Do not allow M mark for $\frac{dy}{dx} =$ linear function drawn			
Graph of $x^2 - \frac{7}{2}x - 3 = 0$			B0M0A0

Q	Answer	Mark	Comments	
21	$2^{(4x+1)(x+3)}$ or $2^{4x^2+13x+3}$	M1	lhs	
	$(2^3)^{x-1}$ or $2^{3(x-1)}$ or 2^{3x-3}	M1	rhs	
	$4x^2 + 12x + x + 3 = 3x - 3$ or $4x^2 + 10x + 6 (= 0)$ or $2x^2 + 5x + 3 (= 0)$	M1dep	oe with expanded brackets dependent on both M marks $4x^2 + 12x + x + 3$ implies first M mark $3x - 3$ implies second M mark	
	$(2x + 3)(x + 1) (= 0)$ or if quadratic formula used $\frac{-5 \pm \sqrt{5^2 - 24}}{4}$ or better	M1dep	oe factorised	
	-1 and $-1\frac{1}{2}$ or $-\frac{3}{2}$ or -1.5	A1		
	Additional Guidance			
	Both answers correct without working will get all marks			M4A1
	Only one solution correct without working scores nothing			M0A0
	No oe for A mark			
Possible to make both sides powers of 8. Mark to the equivalent of this MS				

Q	Answer	Mark	Comments
22	Alternative method 1 – common factor		
	$\frac{\sin x(3\cos x + \sin x)}{4\cos x(3\cos x + \sin x)}$	M1	
	$\frac{\tan x}{4}$	M1dep	
	$\tan x = -\sqrt{3}$	M1dep	could imply first M1dep mark
	120°	A1	
	Alternative method 2 - divide by $\cos^2 x$		
	$\frac{3\tan x + \tan^2 x}{12 + 4\tan x}$	M1	
	$\frac{\tan x(3 + \tan x)}{4(3 + \tan x)}$ or $\frac{\tan x}{4}$	M1dep	
	$\tan x = -\sqrt{3}$	M1dep	could imply first M1dep mark
	120°	A1	
	Alternative method 3 – setting up an equation		
	$4\sin x(3\cos x + \sin x)$ $= -4\sqrt{3}\cos x(3\cos x + \sin x)$ or $3\tan x + \tan^2 x = -3\sqrt{3} - \sqrt{3}\tan x$	M1	
	$\tan x(3\cos x + \sin x)$ $= -\sqrt{3}(3\cos x + \sin x)$ or $\tan x(3 + \tan x) = -\sqrt{3}(3 + \tan x)$	M1dep	
	$\tan x = -\sqrt{3}$	M1dep	could imply first M1dep mark
	120°	A1	
	Additional Guidance		
	Additional solutions		A0
	Untidy mathematical notation can be condoned		
	Correct answer with no working		MOA0

Q	Answer	Mark	Comments
23	$x = 6$ and $y = 32$	B5	B4 $x = 6$ or $y = 32$ and $a = 3$ or $c = 4$ (could be embedded in $3n^2 - 5n + 4$)
			B3 $a = 3$ or $c = 4$ (could be embedded in $3n^2 - 5n + 4$)
			B2 any two different correct equations set up for a combination of a , c , x and y (may include b . Do not include $b = -5$)
			B1 any correct equation set up for a combination of a , c , x and y (may include b . Do not include $b = -5$) or (second difference) $= 18 - 2x$ or $x + y - 32$ (must be correctly simplified)
			Additional Guidance
	Do not accept any equations with n		
	$x = 6$ on answer line with no working and $y \neq 32$		B0
	$x = 6$ and $y = 32$ on answer line with no working		B5